Digital Adaptive Model following Flight Control

Gurbux S. Alag* and Howard Kaufman†
Rensselaer Polytechnic Institute, Troy, N.Y.

Theme

THE inability of simple mechanical linkages to cope with the many control problems associated with high-performance aircraft has led to the present interest in digital fly-by-wire flight control systems.[1,2] Significant among the advantages of digital implementation are the weight and volume savings, ability to design complex controller structures, reliability of digital logic and capability for time-sharing multiple control loops.

Among the complex controller structures which can be considered for implementation in a digital flight computer is an adaptive control system which provides the capability of automatically compensating for parameter and environmental variations that may occur during operation and thus has the potential for providing uniform stability and handling qualities over the complete flight envelope.[3,4]

In view of this potential, a digitial adaptive controller has been developed and applied to the linearized lateral equations of motion for a typical fighter aircraft. The system is composed of an online weighted least-squares identifier, a Kalman state filter, and a single stage, real model following control law. The corresponding control gains are readily adjustable in accordance with parameter changes to ensure asymptotic stability if the conditions for perfect model following are satisfied and stability in the sense of boundedness otherwise.

Contents

The effectiveness of an adaptive control system lies in its ability to rapidly assess the performance and to make desired modifications in the control gains. A procedure fulfilling these requirements is the concept of model following adaptive control in which the design goal is to force the compensated system to duplicate the performance of a reference model. The model following problem can be stated as

Given the aircraft dynamics

$$x_p(k+1) = A_p x_p(k) + B_p u_p(k) \quad (1)$$

where $x_p(k)$ is the aircraft $(n \times 1)$ state vector at sample time k, $u_p(k)$ is the $(m \times 1)$ control vector, A_p and B_p are matrices with appropriate dimensions; find the control $u_p(k)$ such that the process vector $x_p(k)$ approximates "reasonably well" the model state vector $x_m(k)$ defined by

$$x_m(k+1) = A_m x_m(k) + B_m u_m(k) \quad (2)$$

where: $x_m(k)$ is the $(n \times 1)$ model state vector, $u_m(k)$ is the $(m \times 1)$ pilot input vector, A_m and B_m are matrices with appropriate dimensions.

Presented as Paper 74-886 at the AIAA Mechanics and Control of Flight Conference, Anaheim, California, August 5-9, 1974; submitted August 29, 1974; synoptic received February 21, 1975; revision received April 3, 1975. Full paper available from AIAA Library, 750 Third Avenue, New York, N.Y. 10017. Price: Microfiche, $1.50; hard copy, $5.00. **Order must be accompanied by remittance.** This work was supported by NASA Grant No. NGR33-018-183.

Index categories: Aircraft Handling, Stability and Control; Navigation, Control and Guidance Theory.

*Graduate Research Assistant.

†Associate Professor, Electrical and Systems Engineering Department.

Erzberger[5] established the conditions for perfect model following and concluded that if the plant trajectory is to closely follow a particular trajectory of the ideal model, real model following, wherein model states as well as the plant states are used in forming the control law, is needed to realign the trajectories if disturbances cause them to drift apart.

Chan[6] in turn developed a real model following control law which yields perfect model following if Erzberger's conditions are satisfied and an error constrained to lie within controllable bounds, otherwise. Chan's control law is of the form

$$u_p = K(x_m - x_p) + K'_{x_m} x_m + K_{u_m} u_m \quad (3)$$

where

$$K'_{x_m} = (B_p{}^T B_p)^{-1} B_p{}^T (A_m - A_p) \quad (4)$$

$$K_{u_m} = (B_p{}^T B_p)^{-1} B_p{}^T B_m \quad (5)$$

and K is a gain matrix chosen so as to stabilize $(A_p - B_p K)$. Alternately this may be expressed as:

$$u_p = K x_p + K_{x_m} x_m + K_{u_m} u_m \quad (6)$$

where K and K_{u_m} are as defined previously and

$$K_{x_m} = K'_{x_m} + K$$

Based upon the above control law as defined in Eqs. (3-5), the digital adaptive control system shown in Fig. 1 was synthesized. A Kalman filter was used for smoothing the noisy state measurements for subsequent use in control computation, and a weighted least squares identifier[7] was used for tracking the values of the parameters in A_p and B_p which varied with mach number and altitude.

An experimental study had indicated that such a separation of state smoothing from parameter identification was superior to the combined estimation as performed by an extended Kalman filter. Noisy state measurements were used, as is, by the identifier with the resulting parameter estimates being used in the Kalman filter equations.

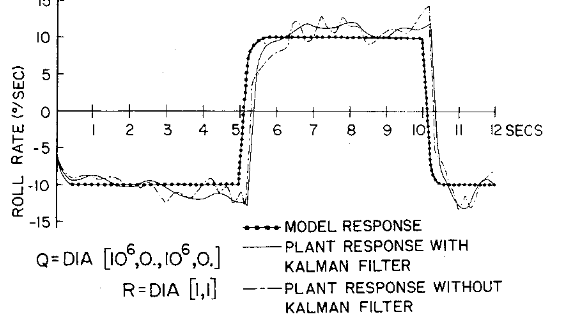

Fig. 1 Digital adaptive controller

As the parameters were tracked by the identifier, the gains K'_{xm} and K_{um} as defined in Eqs. (4) and (5) were computed explicitly by formula evaluation. The gain K was updated by stepping through one or two iterations of a fourth order matrix Riccati equation the solution of which specifies the gain K which stabilizes $(A_p - B_pK)$ [8]. Alternately if K is such that $(A_p - B_pK)$ is stable for all possible values of A_p and, B_p, then no update of K is necessary.

This adpative controller was tested on the linearized lateral equations of motion for a typical fighter aircraft. Thus in Eq. (1)

$$x_p = \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} = \begin{bmatrix} \text{roll rate} \\ \text{yaw rate} \\ \text{side slip angle} \\ \text{roll angle} \end{bmatrix}$$

and

$$u_p = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} \text{aileron deflection} \\ \text{rudder deflection} \end{bmatrix}$$

The elements of A_p and B_p vary with mach number and altitude and for simulation purposes this variation was assumed to be linear between the flight conditions. The typical flight trajectory postulated was defined by citing the order and timing for a typical fighter aircraft to encounter six given flight conditions.

The trajectory corresponded to an initial acceleration from Mach 0.3-0.9 at a very low altitude, a combined climb to 3000 m and acceleration to Mach 1.1, a climb to 15,000 m, a decleration to Mach 0.9, and finally a combined dive to 6000 m and a deceleration to Mach 0.7. The measurement noise characteristics are listed in Table 1.

A sensitivity study conducted in order to determine which parameters were most important for identification, led to the decision to identify the 12 parameters in the first and third rows of A_p and B_p and to set the remaining parameters to values determined by averaging over the six given flight conditions. Using a control sample period of 0.2*S* and a gain adaptation interval of 1.0*S*, state estimates were computed at

Table 1 Measurement noise characteristics

Variable	RMS	Bandwidth
p, roll rate	2 deg/sec	2 Hz
r, yaw rate	0.5 deg/sec	2 Hz
β, side slip	0.3 deg	30 Hz
ϕ, roll angle	0.2 deg	1 Hz

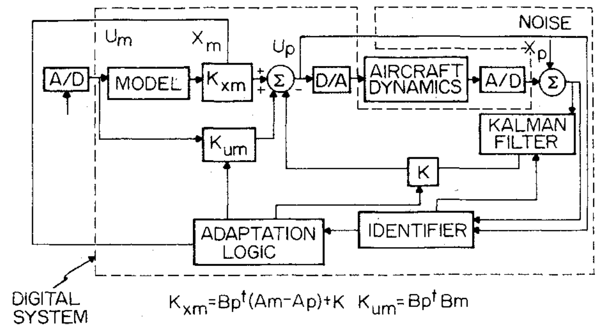

Fig. 2 Roll rate response.

each sample period using the most recently estimated values of the parameters, and then used with the control gains for control computation.

Figure 2 compares the roll rate response with and without state filtering to the model response. The use of state filtering considerably reduces the jitter evident in the response because of the use of the noisey state measurements for control computation. It was further observed that parameter estimates were biased, but did follow the general trend of variation, and that the adapted gain did converge towards the optimal values.

In conclusion, the results indicated the attractiveness of the proposed adaptive controller. Because of the relatively simple gain update logic and the use of the linear parameter and state estimation algorithms the required computation times are such that these functions should be implementable into contemporary minicomputers.

References

[1] Kass, P. J., "Fly by Wire Advantages Explored," *Aviation Week and Space Technology*, July 10, 1972, pp. 52-54.

[2] Sutherland, J., "Fly by Wire Control Systems," *AGARD Conference Proceedings, No. 52, Advanced Control System Concepts*, 1968, pp. 51-72.

[3] Smyth, R. and Ehlers, H. L., "Survey of Adaptive Control Applications to Aerospace Vehicles," *AGARD Conference Proceedings, No. 52, Advanced Control System Concepts*, 1968, pp. 3-13.

[4] Ostgaard, M. A., "Case for Adaptive Control," *AGARD Conference Proceedings, No. 52, Advanced Control System Concepts*, 1968, pp. 15-27.

[5] Erzberger, H., "On the Use of Algebraic Methods in the Analysis and Design of Model Following Control System," TND-4663, July 1968, NASA.

[6] Chan, Y. T., "Perfect Model Following with a Real Model," *Joint Automatic Control Conference Preprints*, 1973, Columbus, Ohio, pp. 287-293.

[7] Young, P. C., "Applying Parameter Estimation to Dynamic Systems—Part I," *Control Engineering*, Vol. 16, Oct. 1964, pp. 119-125.

[8] Moore, B. D. O. and Anderson, J. B., "Linear Optimal Control," Prentice Hall, N.J., 1971, pp. 328-342.